

# Ferrites in Microwave Applications

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*Since Hogan's\* exposition of the extreme usefulness of the microwave Faraday effect numerous other laboratories have begun investigating propagation through ferrites and have made significant contributions to the art. In view of the tremendous interest which is being accorded this work this paper has been prepared to summarize some of the observations and developments to date. The plane wave theory is reviewed briefly with special attention being given to the mechanisms by which power is absorbed by the ferrite. The plane wave theory is then modified to describe various waveguide effects. Finally experimental procedures and results are presented to illustrate the theory and to provide general information regarding the design of devices employing these effects.*

## INTRODUCTION

The ferromagnetic Faraday effect occurs at microwave frequencies as a direct result of the dispersion in permeability which is associated with ferromagnetic resonance. The resonance can be explained most simply by stating that the total magnetization vector of a magnetized ferromagnetic material has associated with it an angular momentum arising from the angular momenta of all of the spinning electrons contributing to the magnetization. Because of this angular momentum (which is directed along the same axis as is the magnetic moment but in the opposite direction) the magnetization vector behaves as a top or gyroscope. If it is displaced from its equilibrium position in a steady magnetic field it will not rotate directly into alignment with the field but will precess about the dc field direction at a frequency determined by the strength of the dc field. In the absence of damping this precession would continue indefinitely, but damping losses are such that the precession will damp out in approximately  $10^{-8}$  sec.

\* C. L. Hogan, The Ferromagnetic Faraday Effect at Microwave Frequencies and Its Applications — The Microwave Gyrator, B.S.T.J., **31**, pp. 1-31, Jan., 1952.

If an alternating field is applied at right angles to the dc field the magnetization will be driven in precession and when the driving frequency coincides with the natural resonance frequency as determined by the strength of the dc field a large amount of power will be absorbed from the driving field. Off resonance the power absorption is small, but the effective permeability seen by the driving field will go through a dispersion such as is exhibited by all resonant systems. With this model in mind we can proceed to discuss the phenomenon of ferromagnetic resonance and the ferromagnetic Faraday effect.

#### INFINITE MEDIUM — LONGITUDINAL FIELD

Polder<sup>1</sup> has shown that because of the gyroscopic nature of the magnetization a tensor permeability is required to relate the magnetic flux density and field intensity vectors in a ferromagnetic medium. At low frequencies the off-diagonal components of this tensor are negligible, and the tensor reduces to the ordinary scalar permeability. At frequencies above about 100 mc. these off-diagonal components can become significant depending upon the magnetic state of the material. When this tensor permeability is introduced into Maxwell's equations and a wave equation is derived for propagation in the direction of the applied magnetic field we find that the normal mode solutions to the wave equation are two circularly polarized waves rotating in opposite directions. A solution in terms of linear polarizations is, of course, possible; but the result is more readily interpreted in terms of the circularly polarized waves. Furthermore, the propagation constant for either circular wave contains a simple scalar permeability, instead of the tensor required to describe the medium in general.

Polder's tensor permeability gives the following relations between  $\mathbf{b}$  and  $\mathbf{h}$  when there is a static magnetic field along the positive  $z$  axis\*.

$$\begin{aligned} b_x &= \mu_0 \mu h_x - j \mu_0 \kappa h_y \\ b_y &= j \mu_0 \kappa h_x + \mu_0 \mu h_y \\ b_z &= \mu_0 h_z \end{aligned} \quad (1)$$

The quantities

$$\mu = \mu' - j\mu'' \quad (2)$$

$$\kappa = \kappa' - j\kappa'' \quad (3)$$

are complex relative diagonal and off-diagonal components of the tensor permeability.

<sup>1</sup> D. Polder, *Philosophical Mag.*, **40**, p. 99, Jan. 1949.

\* Lower case letters are used for RF magnetic quantities.

Equations giving  $\mu$  and  $\kappa$  in terms of the applied magnetic field and the fundamental atomic constants are given by Hogan.<sup>2</sup> These equations show that both  $\mu$  and  $\kappa$  have a resonance at a value of effective internal field given by  $\gamma H = 2\pi f$ . If  $H$  is in ampere turns per meter and  $f$  in megacycles the gyromagnetic ratio is  $\gamma/2\pi = 3.51 \times 10^{-2}$ .

A plane wave travelling in the  $z$  direction in an infinite medium described by equations (1) can be resolved into two counter-rotating circularly polarized plane waves having propagation constants as follows:

$$\Gamma_{+} = j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon(\mu - \kappa)} \quad \Gamma_{-} = j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon(\mu + \kappa)} \quad (4)$$

where the subscripts  $\pm$  refer to positive and negative circularly polarized waves.\* The terms inside parentheses are effective scalar permeabilities completely describing the medium for a circularly polarized wave. Calculated values of these effective permeabilities are plotted as functions of applied field in Fig. 1 for a ferrite operating at three frequencies.

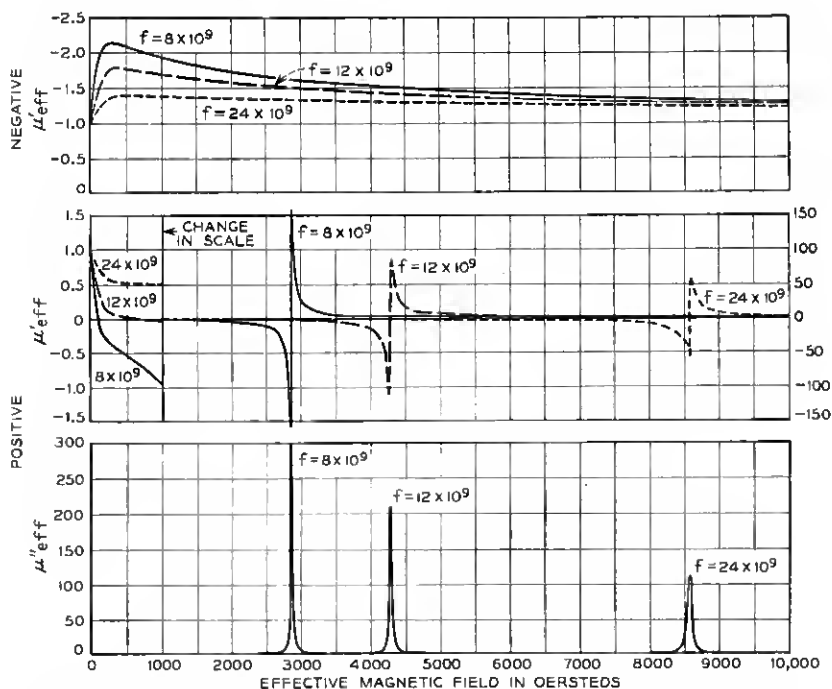


FIG. 1 — Calculated effective permeabilities for positive and negative circularly polarized plane waves computed for three different wave frequencies.

<sup>2</sup> C. L. Hogan, Revs. Mod. Phys., **25**, Jan., 1953.

\* A positive circularly polarized wave is one which rotates in the direction of the positive current producing the dc magnetic field.

From these curves we may calculate the rotation per unit path length, the absorption of the positive component, the net insertion loss and the ellipticity of the resultant wave.

The rotation per unit length is given by

$$\frac{\theta}{l} = \frac{1}{2}(\beta_- - \beta_+) \quad (5)$$

where  $\beta_{\pm}$  are the imaginary parts of the propagation constants,  $\Gamma_{\pm}$ . Let us consider the special case in which the dielectric loss is zero. For convenience we define the complex effective permeabilities seen by the circularly polarized waves as follows:

$$\begin{aligned} \mu_+ &= \mu - \kappa = \mu'_+ - j\mu''_+ \\ \mu_- &= \mu + \kappa = \mu'_- - j\mu''_- \end{aligned} \quad (6)$$

The propagation constants may then be written:

$$\begin{aligned} \Gamma_+ &= \omega\sqrt{\mu_0\epsilon_0} \sqrt{\frac{\epsilon}{2}} [\sqrt{|\mu_+| - \mu'_+} + j\sqrt{|\mu_+| + \mu'_+}] \\ \Gamma_- &= \omega\sqrt{\mu_0\epsilon_0} \sqrt{\frac{\epsilon}{2}} [\sqrt{|\mu_-| - \mu'_-} - j\sqrt{|\mu_-| + \mu'_-}] \end{aligned} \quad (7)$$

It is of particular interest to consider what happens to  $\beta_+$  when  $\mu_+$  becomes zero or negative. If we rewrite the expression for  $\beta_+$ :

$$\beta_+ = \frac{\omega}{c} \sqrt{\frac{\epsilon}{2}} \sqrt{\sqrt{(\mu'^2_+ + \mu''^2_+)} + \mu'_+} \quad (8)$$

we see that, when  $\mu'_+$  is zero or negative,  $\beta_+$  depends primarily on the magnitude of  $\mu''_+$ , for wherever  $\mu''_+$  is negligible,  $\beta_+$  is zero. Furthermore, we see that the attenuation constant,  $\alpha_+$ , given by:

$$\alpha_+ = \frac{\omega}{c} \sqrt{\frac{\epsilon}{2}} \sqrt{\sqrt{\mu'^2_+ + \mu''^2_+} - \mu'_+} \quad (9)$$

becomes dependent primarily upon  $\mu'_+$  when  $\mu'_+$  becomes negative so that we observe a significant attenuation long before  $\mu''_+$  becomes large. In Fig. 2 are shown the rotation of the plane of polarization of the linearly polarized wave and the absorption of the positive circularly polarized component of the wave. The dielectric constant of the ferrite was assumed to be 9.0, a typical value for many ferrites.

From these curves it is evident that the wave will be elliptically polarized whenever the effective field is large enough to make  $\mu'_+$  zero or

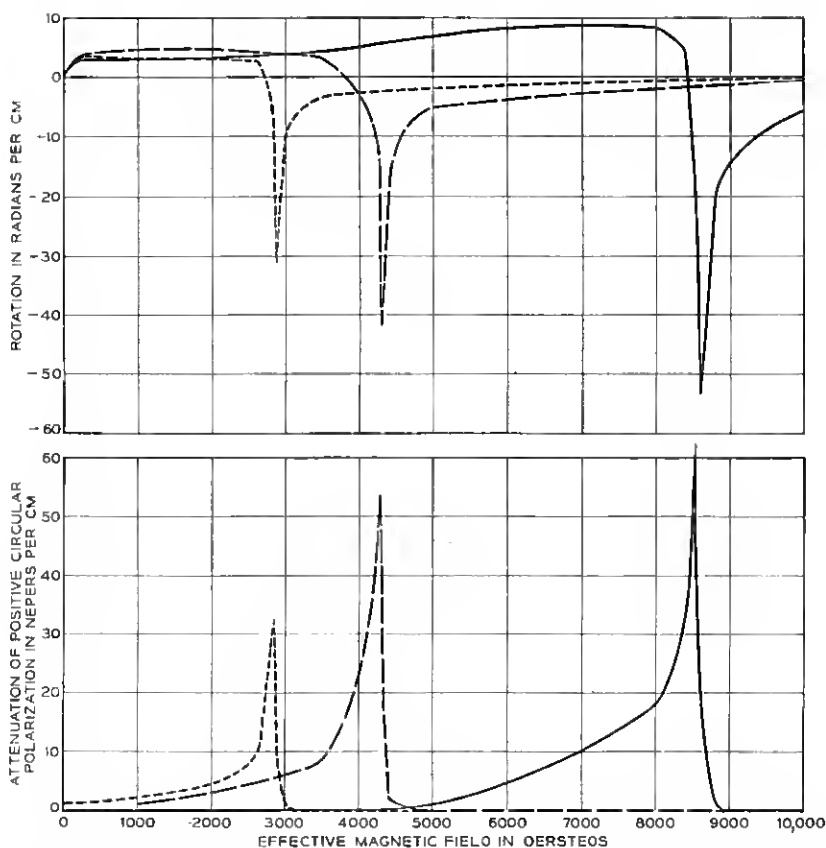


Fig. 2 — Rotation of the linearly polarized plane wave and absorption of positive circularly polarized component versus effective static field computed from data of Fig. 1.

negative, the amount of ellipticity depending in part upon the distance the wave has traveled in the ferrite medium.

#### PLANE WAVE, TRANSVERSE FIELD

If a wave is propagated in either the  $x$  or  $y$  direction when the dc field is in the  $z$  direction then the wave equation has two orthogonal solutions representing linearly polarized waves. One of these is polarized with the electric vector parallel to the applied dc field and the other has the magnetic vector parallel to the applied dc field. When the magnetic

vector of the wave is parallel to the magnetization the torque on the electrons is zero and the wave sees an isotropic dielectric medium with relative permeability equal to unity. However, when the electric vector is parallel to the magnetization the magnetic vector is at right angles to it and can set the electrons into precession.

Consider a wave propagating in the  $x$  direction in an infinite medium magnetized in the  $z$  direction. Let this wave be polarized so that it has components  $E_y$  and  $h_y$ . When this wave enters the magnetized medium,  $h_y$  exerts a torque on the magnetization vector  $M$  causing it to precess about the  $z$  axis. This results in both an  $m_y$  and an  $m_x$  component of alternating magnetization. There is, however, no component of  $\mathbf{b}$  in the  $x$  direction because of internal demagnetizing fields arising from an effective volume distribution of magnetic charge as shown in Fig. 3. Such a volume distribution of magnetic charge arising from the periodic reversal in phase of the driving magnetic field is propagated through the medium at the velocity

$$v = \frac{c}{\sqrt{\mu\epsilon}}$$

An instantaneous picture of this distribution is shown in Fig. 3. The magnetic poles set up a magnetic field in the  $x$  direction throughout the material. This field is commonly called a demagnetizing field and for

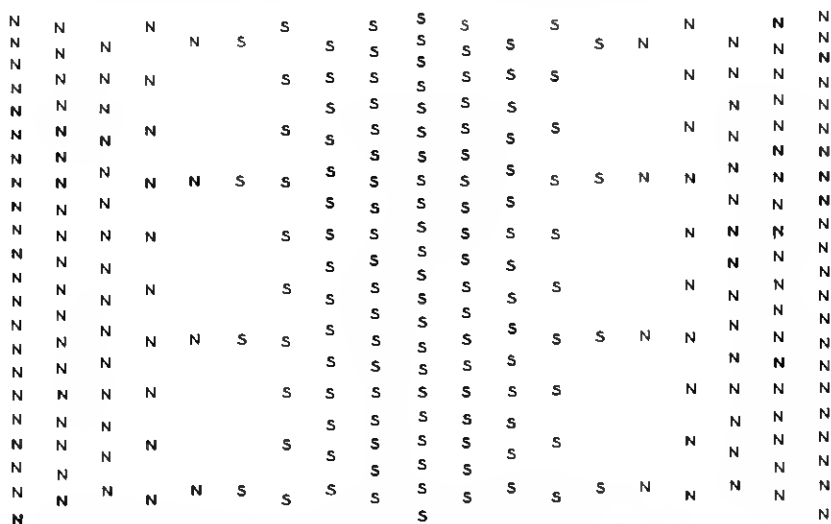


FIG. 3 — Effective volume distribution of magnetic poles arising from phase reversals in transversely magnetized infinite medium.

this particular case it can easily be shown that at every point in the material this field is given by\*

$$h_x = -m_x$$

and by definition

$$b_x = \mu_0(h_x + m_x) = 0$$

In one sense this wave is no longer a plane wave as it has a component of  $\mathbf{h}$  in the direction of propagation. However, the electric field,  $E$ , and the magnetic flux density,  $\mathbf{b}$ , are unchanged and remain the same as in a normal plane wave.

The solution to the wave equation for the foregoing case yields a propagation constant

$$\Gamma = j\omega\sqrt{\mu_0\epsilon_0} \sqrt{\frac{(\mu^2 - \kappa^2)\epsilon}{\mu}} \quad (10)$$

in which the effective relative permeability of the medium is

$$\mu_{\text{eff}} = \frac{\mu^2 - \kappa^2}{\mu} \quad (11)$$

The real and imaginary parts of this expression are plotted in Fig. 4 for three frequencies.

These curves have the same general shape as those for the *positive* circular component of the wave propagated along the dc field direction, but here they apply to the entire linearly polarized wave. Again we have the possibility of zero or negative permeability. In the region just above resonance the real part of the permeability takes on large values and maintains these even after the absorption curve is nearly zero. This suggests that it is possible to adjust the permeability to equal the dielectric constant of the material so that the medium matches free space perfectly. The medium then can be used as a switch by changing the field from the point where  $\mu_{\text{eff}} = 0$  to the point where  $\mu_{\text{eff}} = \epsilon$ .†

In the region between zero applied field and saturation where the curve levels off, the effective permeability changes almost linearly. In the

\* We follow Stratton in making  $M$  an  $H$ -like quantity rather than  $B$ -like.

† In a waveguide  $\mu_{\text{eff}}$  must be adjusted to satisfy the condition that

$$1 = \mu_{\text{eff}} \sqrt{\frac{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}{\mu_{\text{eff}}\epsilon - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

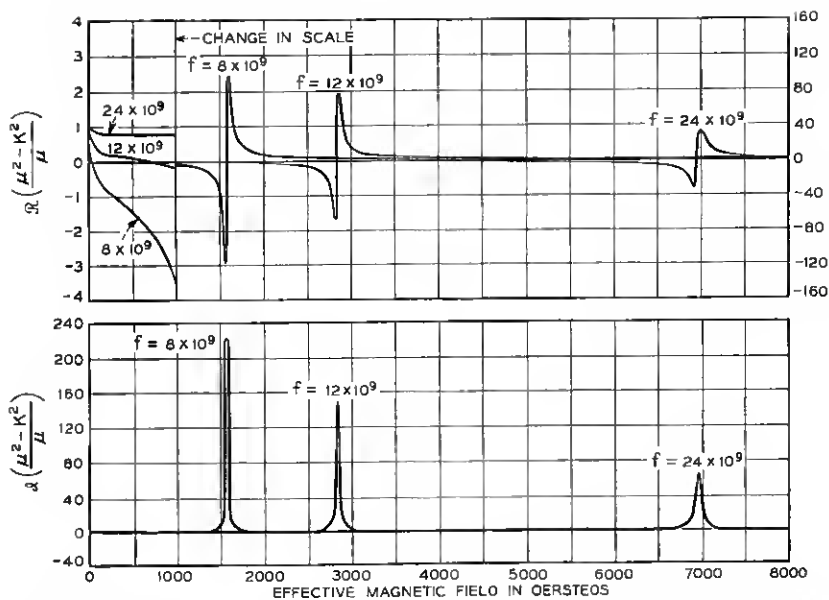


FIG. 4 — Effective permeability seen by a plane wave in a transversely magnetized infinite medium. Real part above and imaginary part below.

absence of dielectric loss the propagation constant of the wave is given by:

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon} \sqrt{\frac{(\mu^2 - \kappa^2)}{\mu}} = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon} \sqrt{\mu_{\text{eff}}} \quad (12)$$

Thus a variable phase delay is obtained by controlling the magnitude of the applied field. This delay is not necessarily accompanied by a change in attenuation so that an ideal phase shifter can be made using this effect. There are numerous other applications which can be made of the longitudinal and transverse applied field phenomena. Many of these will be discussed later in this article.

#### LOSS MECHANISMS IN FERRITES AT MICROWAVE FREQUENCIES

Neglecting dielectric losses, the plane wave theory predicts almost no loss at all for a negative circularly polarized wave and a single absorption line for a positive C.P. wave. In practice a more complicated behavior is observed, and to facilitate the discussion we show in Fig. 5 typical loss characteristics superposed upon the theoretical loss curve of Fig. 1. We will enumerate the main points of interest before proceeding with the discussion. The specific differences in behavior are:



Curve A. Broad resonance absorption line.

Curve B. A loss which disappears when the material is magnetized, called "Low Field Loss".

Curve C. Loss which goes to zero for one component and rises for the other.

Curve D. A loss which appears to be independent of magnetic field over a wide range and can be related to the dielectric loss tangent of the material, hence called dielectric loss.

Curve E. Higher order modes causing erratic variations in loss.

Curve F. Double peaks due to "Cavity Resonances".

Qualitative and semiquantitative explanations have been developed to explain all of these phenomena. Some of them follow from a simple extension of the plane wave theory and the rest are based upon considerations of the special case of a partially filled waveguide.

#### *Curve A, Fig. 5*

Associated with the precessional resonance there is a damping term by which power is dissipated in the lattice. The exact nature of this damping term is not fully understood, and measured line widths are always greater than those predicted by present theory. Nevertheless we have at our disposal empirical damping constants which can be used to predict resonance absorption losses as was done in the calculation of the curves of Figs. 1 and 4.

These apply, however, only to small ellipsoidal samples which are ground from single crystal ferrites. In polycrystalline ferrites the absorption line is generally broader for three reasons, namely; crystalline anisotropy, strain anisotropy and varying internal demagnetizing fields due to the variety of shapes of the constituent crystallites.

Many ferrites have a high crystalline anisotropy which behaves in

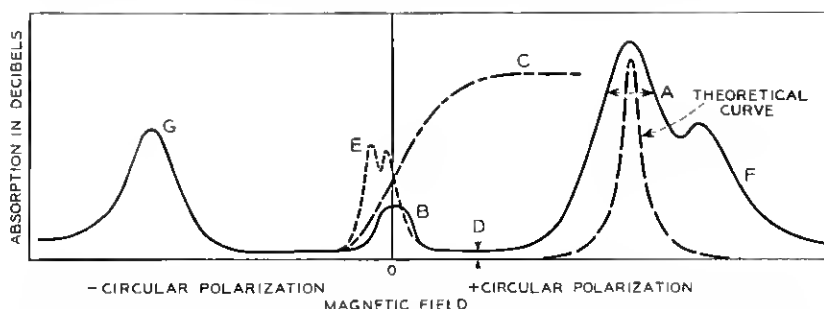


FIG. 5 — Typical loss characteristics encountered in the measurement of various ferrite samples in cylindrical waveguides with longitudinal static field.

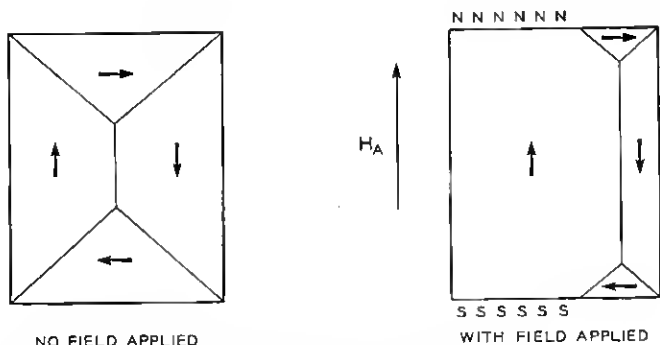


FIG. 6 — A typical domain wall pattern showing the movement of the wall in response to the application of an external magnetic field.

many ways like an internal field tending to keep the magnetization of the constituent crystallites along one of the axes of easy magnetization. In most ferrites there are four such axes and since the magnetization can be in either direction along any one of these, there are eight directions of easy magnetization. When a field is applied the effective internal field is roughly the vector sum of the applied field and the anisotropy field associated with the nearest axis. In a polycrystalline ferrite composed of many randomly oriented crystallites it is evident, therefore, that the internal field varies from point to point in the body so that the resonance absorption line is broadened by an amount proportional to the magnitude of the anisotropy field. A similar broadening can arise from magnetostriction due to fields arising from varying strains throughout the polycrystalline ferrite, and non-uniform internal demagnetizing fields due to the shape of the constituent particles or crystallites can likewise broaden the resonance line. Since a broad resonance line results in ellipticity of the transmitted wave it is desirable to use a ferrite having low anisotropy.

#### *Curve B, Fig. 5*

Frequently a loss is observed at low fields as indicated in Fig. 5 by Curve B. Neglecting waveguide effects this hump is symmetrical<sup>3</sup> so that it evidently depends on a phenomenon which affects both circular components equally. It is generally agreed that it depends upon the existence of domain walls within the material since it usually disappears as soon as the body is magnetized. However, there is some question as to the specific mechanism involved.

<sup>3</sup> Fox and Weiss, *Revs. Mod. Phys.*, **25**, p. 262, Jan., 1953.

A ferromagnetic crystal consists entirely of regions called domains which are completely magnetized along one of the directions of easy magnetization. In general the direction of magnetization of these domains is varied in an orderly manner as shown in Fig. 6 so that the energy of the crystal as a whole is a minimum. In the region between adjacent domains there is a (usually) narrow wall in which the magnetization goes through a gradual change in direction from that of one domain to that of the other. When an external field is applied the magnetization of the crystal is increased by the growth of some domains at the expense of their neighbors. When the crystal is saturated substantially all of the walls have disappeared and the material behaves as a single large domain.

There are currently two proposed mechanisms by which these domain walls could cause a loss at low fields. Becker and Döring<sup>4</sup> have shown that there can be associated with the motion of a domain wall either relaxation or resonance frequencies. Galt<sup>5</sup> has measured relaxations in a single crystal of magnetite at 3,000 cps and in a single crystal of nickel ferrite at approximately 2.5 mc and has presented a rather convincing argument that these are due to domain wall motion. In general the *relaxation* frequency would be expected to occur far below the microwave frequencies, but resonances could conceivably occur at microwave frequencies and could be quite broad. Until recently no other theory had been advanced which would explain the losses so often observed at low fields, and these were, therefore, attributed to a high frequency domain wall resonance.

There is a more satisfactory explanation which has recently been stated in different ways by Rado<sup>6</sup> and by Smit and Polder.<sup>7</sup> Rado has observed a resonance absorption in the microwave region with zero applied field and has shown from temperature dependence that the frequency of this resonance depends upon the saturation magnetization and the crystalline anisotropy of the ferrite. Smit and Polder have presented a model by which we can see how both of these quantities can enter to produce a loss at low fields.<sup>7</sup> We consider an ellipsoidal crystallite as shown in Fig. 7. The domain structure shown is one which could exist in some crystallites in a polycrystalline ferrite.

The magnetization in domains numbered 1 will respond to right circular polarization and the others to left circular. In other words a wave rotating clockwise is positive circularly polarized in domains one while a

<sup>4</sup> Becker and Döring, *Ferromagnetismus*, Springer, Berlin, 1939.

<sup>5</sup> J. K. Galt, *Phys. Rev.*, **85**, Feb. 15, 1952.

<sup>6</sup> G. T. Rado, R. W. Wright, et al., *Phys. Rev.*, Nov. 1952.

<sup>7</sup> D. Polder and J. Smit, *Revs. Mod. Phys.*, **25**, pp. 89-90, Jan. 1953.

wave rotating *counterclockwise* is positive circularly polarized in domains numbered 2. In the absence of any other effects the resonance frequency of all of these domains would be determined by the anisotropy field. However, if we excite both circular polarizations simultaneously and if the relative phase of the two circular polarizations is as shown in Fig. 7(a) poles will be set up at the domain walls as indicated in the figure. The demagnetizing fields associated with these will cause the resonance for both circular components to occur at a frequency given by:

$$f = \gamma H_{\text{eff}} \cong \gamma M_z \quad (13)$$

On the other hand, if the phase of the two circular polarizations is as shown in Fig. 7(b), no poles will be set up on the walls and the resonance will be determined primarily by the anisotropy field.

$$f = \gamma H_{\text{eff}} \cong \gamma H_{\text{anis}} \quad (14)$$

These two examples of the relative phase of the circular waves correspond to linear polarizations in the  $x$  and  $y$  directions respectively.

This simplified derivation gives the maximum and minimum frequencies at which resonances can occur. In a material containing a large number of randomly shaped and randomly oriented crystallites, resonances can occur at all frequencies between these limits. Most ferrites have values of  $M_s$  between 80,000 and 240,000 and anisotropy fields which probably range from 8000 to 80,000 amp. turns/meter with per-

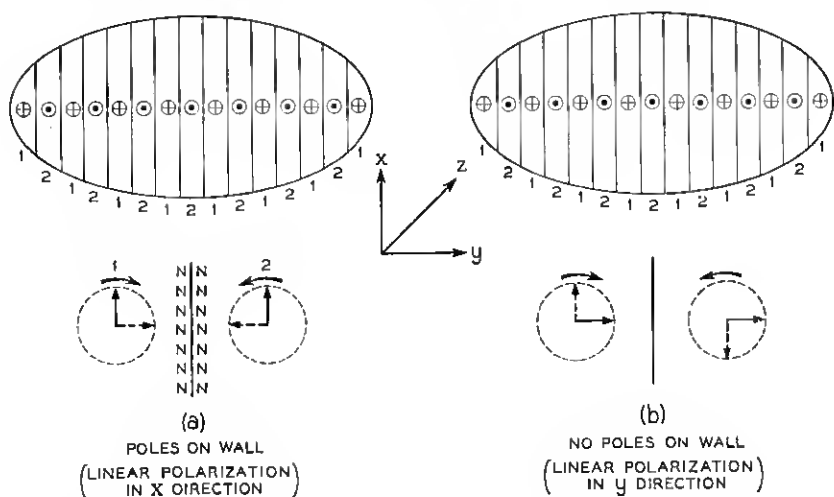


Fig. 7 — Model used by Smit and Polder to illustrate their theory for the "low-field loss."

haps a few which are higher still. Therefore, we have as typical frequencies

$$f_1 = \gamma H_{\text{anis}} \cong 300 \text{ to } 3,000 \text{ mc} \quad (15)$$

and

$$f_2 = \gamma M_s \cong 3,000 \text{ to } 9,000 \text{ mc.} \quad (16)$$

It is evident that at 9,000 mc only that loss associated with  $M_s$  will contribute to the "Low Field Loss" and since the mechanism depends upon the existence of domain walls it will disappear when the material is saturated in agreement with our observations.

#### *Curve C, Fig. 5*

Some ferrites exhibit a low-field loss which disappears at saturation for only the negative component and increases for the positive component. This is thought to be due to an effective anisotropy field in the material. In order for such behavior to be present at 9,000 mc, however, the internal field must be of the order of 240,000 ampere turns per meter. While such a value of crystalline anisotropy might be found in cobalt or other high anisotropy ferrites it appears to be somewhat too high for a nickel-zinc ferrite such as that in which this characteristic was first observed. However, high internal fields could result from demagnetizing effects similar to those discussed under item B but differing in that the poles are set up on nonmagnetic grain boundaries instead of domain walls. These, of course, would persist when the body is saturated, but there would be loss for only one circular component inasmuch as all of the crystallites are then magnetized in the same direction. Such a loss characteristic can be quite useful where one wishes to absorb one circular component selectively without the necessity for applying a large dc field.

#### *Curve D, Fig. 5*

Dielectric losses are present in all of the ferrites which have been made to date, although in some materials this loss is very low. Low dc conductivity in itself is not a sufficient criterion of the dielectric properties of a material as some ferrites appear to consist of conducting regions surrounded by an insulating matrix, and these have fairly high loss tangents at microwave frequencies.

The major mechanism of dielectric loss involves the exchange of electrons between ions in the crystal lattice. It has been found that the presence of ions of the same metal in different valence states on the same

lattice site gives rise to high conductivity and hence high dielectric loss. Conversely, when a ferrite is carefully prepared so that all of the constituents are present in exactly stoichiometric proportion, and when the possibility of multiple valence states is eliminated the conductivity is very low. To illustrate this point a series of measurements is reported in which the iron content of the ferrites was carefully varied about stoichiometry in a nickel-zinc ferrite. These measurements are discussed at a later point in this paper.

A ferrite is made by reacting a mixture of metallic oxides at a temperatures below the melting point of these oxides. As the oxides react a new crystal structure is evolved in which the metallic ions occupy positions in the interstices of a close-packed oxygen lattice. There is a very well authenticated theory due to Neel<sup>8</sup> explaining the way in which the spin orientations of the ions are distributed in the two types of lattice site which exist in the Spinel oxygen lattice. Whenever metal ions in more than one valence state occupy the same type of site, e.g., the octahedral position, there is a possibility for the easy transfer of an electron from one to the other since the crystal structure is unchanged by the transfer.<sup>8</sup> In the case of nickel ferrite which has the composition  $\text{NiOFe}_2\text{O}_3$  an excess of iron will tend to replace some nickel atoms by entering the lattice in the divalent state. Since the remainder of the iron is trivalent, comparatively high conductivity is observed. The problem of producing ferrites with extremely low dielectric losses appears to be fairly well understood and is progressing satisfactorily. By choosing the proper set of metal ions to insure the absence of multiple valence states and by maintaining the proper oxygen stoichiometry one may be able to achieve loss tangents as low as 0.001. The subject of dielectric losses is well covered in the literature.<sup>9, 10</sup>

### *Curves E, F and G, Fig. 5*

The loss mechanisms indicated in Fig. 5 by Curves E and F all arise from the particular behavior of ferrites in waveguides as differentiated from the plane wave theory.

For example, the erratic behavior indicated by Curve E has been shown to be due to the presence of higher order modes in the ferrite region in a waveguide, and the subsidiary hump on the absorption Curve F has been shown to be a "cavity resonance" which is strongly dependent

<sup>8</sup> L. Neel, *Physica*, **16**, pp. 350-53, 1950, and *Zeit. Anorg. Chem.*, **262**, pp. 175-184, 1950.

<sup>9</sup> E. J. W. Verwey and J. H. DeBoer, *Rec. des Travaux Chimiques des Pays-Bas*, **55**, pp. 531-54, 1936.

<sup>10</sup> E. J. W. Verwey et al., *Phillips Res. Repts.*, **5**, pp. 173-187, 1950.

upon the diameter of the ferrite cylinder and upon the guide wavelengths but not upon the length of the cylinder.

Other waveguide effects causing anomalous loss behavior, such as shown by curve G, have been discussed by Fox and Weiss<sup>11</sup> and will be treated by them in greater detail in a forthcoming publication.

In order to discuss these effects more fully we must examine the modifications of the plane wave theory which must be made to explain the behavior of a ferrite in a waveguide.

#### WAVEGUIDE THEORY, LONGITUDINAL FIELD

When a piece of ferrite is placed in a waveguide and magnetized it is necessary to modify the foregoing plane wave theory to describe the behavior of a wave passing through the ferrite. Because of the anisotropic nature of the magnetized ferrite it is necessary to obtain a solution to the specific problem of the waveguide containing the ferrite. When the magnetization of the ferrite precesses about the applied dc field it sets up components of  $\mathbf{h}$  which do not exist in any of the classical modes, and unless one can deal with small perturbations the solution becomes quite involved.

The modes which can exist in the ferrite will often resemble the classical modes so that for convenience we will refer to them as modified TE or TM modes. Suhl and Walker<sup>12</sup> have obtained solutions for the case of a cylindrical waveguide completely filled with a magnetized ferrite, and they have shown that the modified dominant  $TE_{11}$  mode behaves much like the plane wave in the region of small fields but that the behavior of the TM modes cannot be approximated by a simple extension of the plane wave theory. A waveguide large enough to support the dominant mode when filled with air will, when filled with ferrite, support three or four higher order modes including some of the modified TM modes. In this case, it is possible to have several present at the same time with the result that observations of rotation, loss and ellipticity are almost impossible to interpret. Accordingly, we should reduce the size of the waveguide in the ferrite-filled region, and this involves the creation of discontinuities in the waveguide. This is not always necessary, however, because higher order modes will not always be set up in the ferrite even though the waveguide is large enough to propagate them. If care is taken to avoid geometries which favor a given mode the prob-

<sup>11</sup> A. G. Fox and M. T. Weiss, *Revs. Mod. Phys.*, **25**, p. 262, Jan., 1953.

<sup>12</sup> A preliminary report of this work has been published in the form of a Letter to the Editor by H. Suhl and L. R. Walker in *Phys. Rev.*, **86**, p. 122, 1952. A more detailed account is scheduled for publication in the *J. Appl. Phys.*

ability of its occurrence will be greatly reduced. In particular it has been found that a flat-ended cylinder completely filling the waveguide can be introduced into the full-sized waveguide without mode complications, but Fox and Weiss<sup>13</sup> have shown that putting conical tapers on the ends will favor the establishment of the modified  $TM_{11}$  mode. In most applications of the Faraday effect the ferrite element is in the form of a very thin pencil at the center of the waveguide so that the mode problem is greatly simplified, but in order to obtain quantitative fundamental information about ferrites themselves it is often necessary to work with a completely filled waveguide. In such cases considerable care must be taken to insure the validity of the measurements.

One method of making impedance measurements which has been used successfully by H. Suhl is to cut a shallow longitudinal slot in the cylinder of ferrite and to make standing wave measurements directly in the medium. Because the slotted section is filled with ferrite the size of the waveguide can be reduced to insure the presence of a single mode. This is restricted to unmagnetized materials as rotation of the plane of polarization would result in radiation by the slot.

Another suggested procedure is to make a transformer from full-size rectangular waveguide to circular waveguide of diameter equal to  $d/\sqrt{\epsilon}$  where  $d$  is the diameter of a dominant-mode air filled pipe and  $\epsilon$  is the relative dielectric constant of the ferrite. This transformer can be treated as a four terminal impedance transformer and its network impedances can be determined by measurement. Impedances measured in the air filled guide will have to be transformed through this network to obtain the true impedances of the ferrite-filled guide, but this can be done if the need for the measurements warrants such effort.

An exact solution of the partially filled waveguide is considerably more difficult to obtain than the solution for a completely filled waveguide. Yet this geometry is the one usually used in most practical applications of the Faraday effect. In the absence of an exact solution one must develop simple physical explanations based upon plane wave theory plus intuition for numerous observed phenomena. A theory for the partially filled longitudinally magnetized waveguide can easily be developed from two simple observations. First, we consider the circular components of the wave separately and observe that each sees an effective scalar permeability which is a weighted average of the permeability of the pencil for that component and that of the surrounding medium, and second we postulate that a small enough pencil will not act as a dielectric rod waveguide and will merely create a small perturbation of

<sup>13</sup> A. G. Fox and M. T. Weiss, *Revs. Mod. Phys.*, **25**, p. 262, Jan., 1953.



the original mode. When the above assumptions are valid the plane wave theory can be extended easily to explain loaded waveguide behavior. In the plane wave case it was shown that a negative effective permeability results in attenuation of the positive circularly polarized wave. Quite a different result is observed in waveguides containing very small cylinders of ferrite. It appears that if the cylinder is small enough not to act as a dielectric waveguide then the negative permeability inside the rod simply is averaged with the permeability of the surrounding region so that the rotation curve (which depends upon the difference between the square roots of the permeabilities seen by the two circular components of the wave) follows the dispersion curve of the permeability of the positive circularly polarized wave, even following the pattern of the permeability when it is negative.

In Fig. 8 are shown measured curves of the rotation of the wave and the absorption of the positive circularly polarized component of the wave as functions of applied dc field for comparison with Figs. 1 and 2. To amplify our arguments we point out that the propagation constant in a waveguide containing a very small pencil of ferrite is of the form:

$$\beta_{\pm}^2 = \beta_0^2 + A \frac{r_1^2}{r_0^2} \left[ \frac{\mu_{\pm} - 1}{\mu_{\pm} + 1} \beta_0^2 - \beta_0^2 \frac{\epsilon - 1}{\epsilon + 1} \right] \quad (17)$$

where  $\beta_0$  is the propagation constant of the empty guide

$r_1$  is the radius of the ferrite cylinder

$r_0$  is the radius of the waveguide

$\epsilon$  is the dielectric constant of the ferrite

and  $\mu_{\pm}$  are the effective complex permeabilities ( $\mu \pm k$ )

$A$  is a constant  $\cong 3.2$

We see that the expression within the brackets is finite for all values of  $\mu_{\pm}$  except  $\mu_+ = -1 + j0$ . Accordingly if the damping parameter (or line width) is large enough to insure that  $\mu_+$  can never take on this value, there will always be a cylinder radius,  $r'$ , for which the perturbation term is small relative to  $\beta_0^2$ . However, the cylinder diameter does not have to be very large before the above arguments fail and the rotation and loss behavior become quite different.

The cutoff wavelength of the  $TE_{11}$  mode in round guide is

$$\lambda_c = 0.1708 d \sqrt{\mu\epsilon}$$

where  $d$  is the diameter in centimeters of the waveguide and  $\mu$  and  $\epsilon$  are the effective relative permeability and dielectric constant of the medium contained therein. When a cylinder of ferrite is placed at the center of

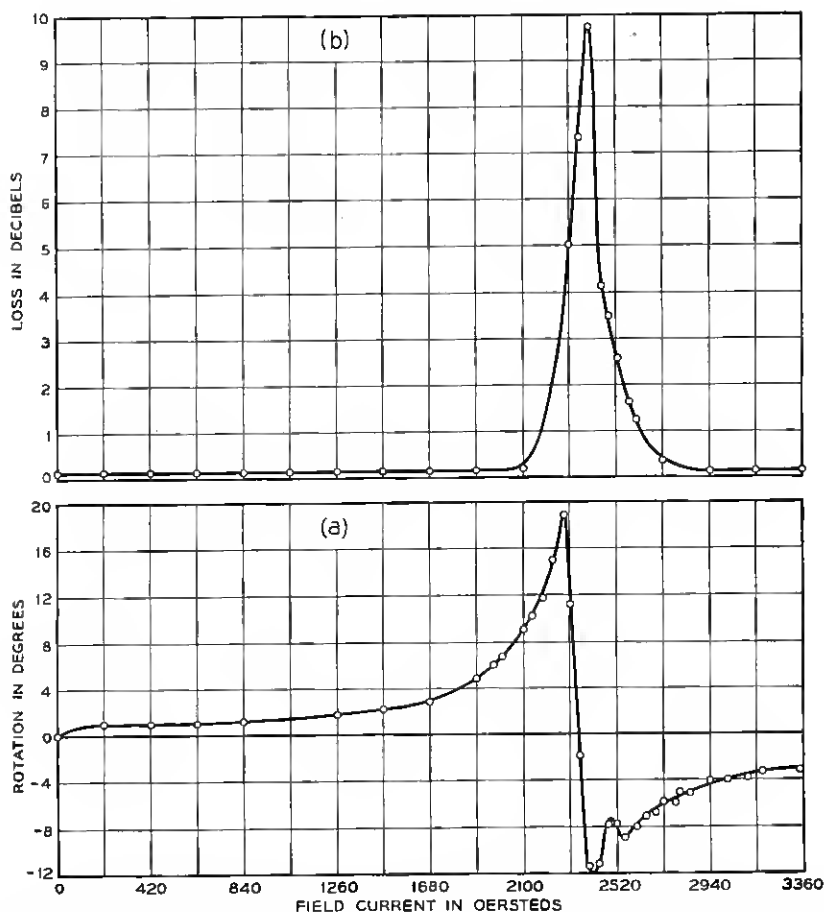


FIG. 8 — Rotation of the plane of polarization and absorption of the positive component as functions of applied dc field for a wave propagating through a waveguide containing a very small cylinder of ferrite. The inflection of 2,400 oersteds is due to a "cavity resonance" as discussed later.

the waveguide the cutoff wavelength is increased because the effective value of  $\epsilon$  is increased. If the ferrite cylinder is quite small this alteration will be unimportant, but if the diameter of the rod exceeds about one-quarter that of the air filled dominant mode guide this mechanism can lead to the existence of higher order modes for the negative component. When this happens the plane wave theory obviously cannot be expected to apply, for the presence of multiple modes in the propagation of either component will introduce an additional variable not present in the plane

wave theory. Experimentally one will observe very erratic and frequency-dependent behavior under these conditions.

#### WAVEGUIDE THEORY — TRANSVERSE FIELD

A waveguide, either round or rectangular, filled with ferrite and magnetized by a field parallel to the *electric* vector of the dominant mode will exhibit a behavior qualitatively the same as described in the plane wave theory of the transverse field. In fact, it has been shown that in a rectangular guide all of the  $TE_{on}$  modes can exist with only slight modification.<sup>14</sup> That this result is probable may be seen from the fact that the precessing magnetization vector sets up components of  $\mathbf{h}$  in the  $x$  and  $y$  directions when the applied field is in the  $z$  direction, and both of these components normally exist in the  $TE_{on}$  modes. The primary modification of the mode arises from RF demagnetizing fields in the ferrite. Because of this modification it is extremely difficult to match the boundary conditions for normal incidence at an interface between the ferrite and air in the waveguide. An infinite series of modes is actually required, but in practice the mismatch due to magnetic effects is usually not very large. If one matches the dielectric constant by means of tapered dielectric horns the remaining mismatch is slight except where  $\mu_{eff}$  approaches zero and at resonance.

While the completely filled waveguide magnetized by a transverse magnetic field parallel to the electric vector of the wave will exhibit a reciprocal behavior in respect to phase change and attenuation, an interesting and potentially useful modification of these effects occurs when a small piece of ferrite is located asymmetrically in a waveguide. Chait and Sakiotis<sup>15</sup> of the Naval Research Laboratory and Turner of the Holmdel laboratory of Bell Telephone Laboratories have independently observed a phase shift which is dependent upon the direction of propagation of the wave, and a simple explanation of this effect has been made by Turner<sup>16</sup> and by Kales.<sup>15</sup> Suhl and Kales<sup>15</sup> have shown the theoretical validity of this explanation. The idea can be demonstrated by consideration of the field configuration shown in Fig. 9.

An observer at the point  $P$  will see an  $\mathbf{h}$  field which is elliptically polarized in a plane normal to the direction of  $H_A$ . The sense of the rotation of the larger circular component of the ellipse will depend upon the direction of propagation of the wave. Thus for *one* direction the major

<sup>14</sup> A. A. van Trier, Paper presented orally at meeting of Amer. Phys. Soc., Washington, D. C., April, 1952.

<sup>15</sup> M. L. Kales, H. N. Chait and N. G. Sakiotis, Letter to the Editor, J. Appl. Phys., June, 1953.

<sup>16</sup> E. H. Turner, Letter to the Editor, Proc. I. R. E., **41**, p. 937, 1953.

part will be a *positive* circular polarization with respect to the applied  $H_A$  and will experience a decrease in  $\mu'$  and for the other direction of propagation the  $\mathbf{h}$  field will be primarily *negative* circularly polarized and the wave will experience an increase in  $\mu'$ . Since the  $\mathbf{h}$  field is linearly polarized in the transverse plane at the center of the guide and is linear in the longitudinal direction at the edge of the guide it is evident that there is a point in between where the differential phase shift is maximum. Suhl has shown that this point always occurs halfway between the guide wall and the center regardless of the proximity of cut-off.

Obviously one has merely to adjust the length of the sample and the strength of the magnetic field so that the differential phase shift is  $180^\circ$  and he will have a gyrator. Then it is an easy matter to design a circulator, isolator, or any of the numerous devices depending upon the gyrator action.<sup>17</sup>

#### EXPERIMENTAL PROCEDURES AND RESULTS

In order to verify the theory and to determine the optimum performance obtainable in microwave devices employing the Faraday Rotation an extensive measurement program has been set up. Information of both a fundamental and of a practical nature is obtained through a variety of measurements. From a practical point of view we are interested

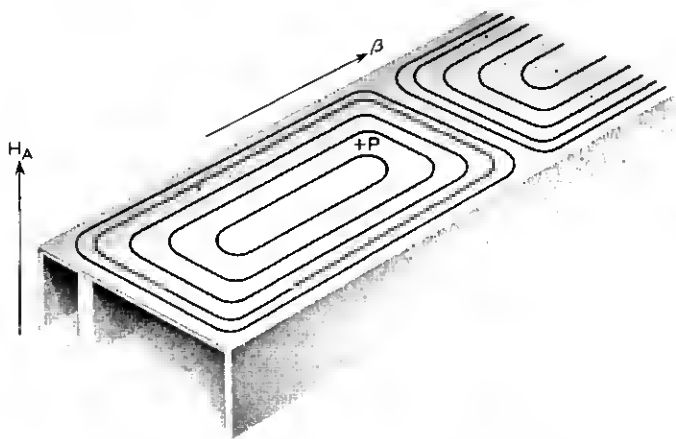


FIG. 9 — Rectangular waveguide containing assymmetrically located ferrite slab. Magnetic lines of force are shown on the top wall of the guide.

<sup>17</sup> C. L. Hogan, The Ferromagnetic Faraday Effect at Microwave Frequencies and Its Applications — The Microwave Gyrator, B.S.T.J., **31**, pp. 1-31, Jan., 1952.

in obtaining a ferrite which gives the maximum rotation per unit loss, and at the same time we are interested in obtaining such fundamental information as the mechanisms of loss, the effect of composition, and the relationship between the various physical properties of the ferrite crystals and their microwave behavior. As a corollary interest we wish to relate the microwave behavior of ferrites to their low frequency performance. For convenience most of the microwave measurements have been made in the 9,000 mc X-Band region, but 4,000, 24,000 and 48,000 mc measurements have been carried out by others in Bell Laboratories and some of their findings will be reported here for comparison with those obtained at X-Band.

In the most common procedure the Faraday Rotation and attenuation of a linearly polarized wave and the ellipticity of the resultant wave are measured as a function of the intensity of the applied longitudinal magnetic field. The experimental equipment used in these measurements is shown in Fig. 10(a). A variety of sample shapes ranging from cylinders completely filling the waveguide to very small cylinders suspended along the axis have been measured. From these data the absorption of the positive circular component may be obtained if one assumes that the attenuation of the negative component remains constant as predicted in the plane wave theory. This is not always the case, however, due to the mode configurations sometimes present, and for this reason an alternate

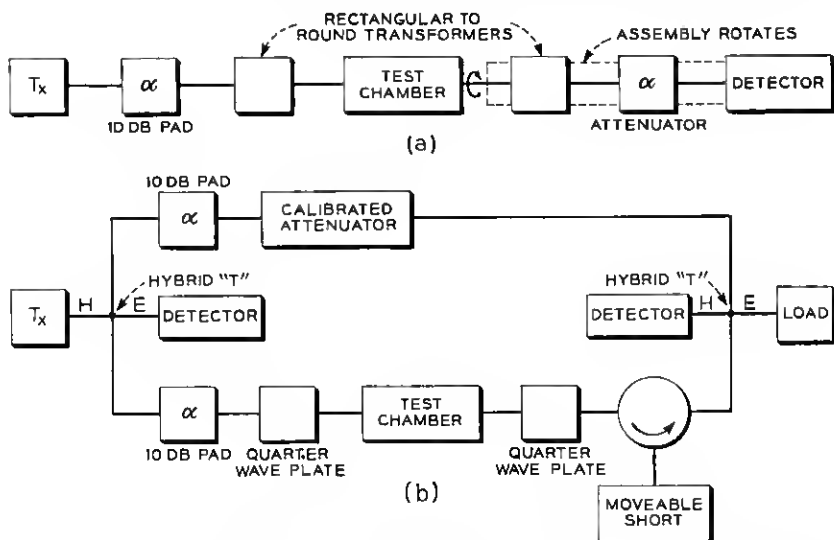


FIG. 10 — Block diagrams of the two most commonly used measuring set-ups.

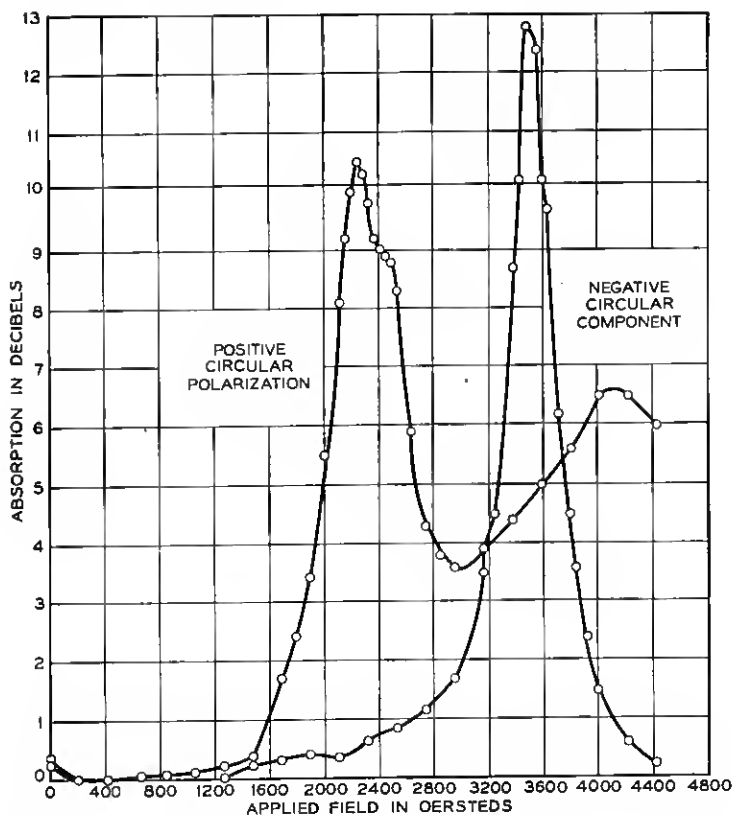


Fig. 11 — Absorption versus applied field for positive and negative circular polarizations. Ferrite cylinder diameter was 0.125".

procedure is often used. This consists simply of exciting the circularly polarized components separately and measuring the phase shift and attenuation of each of these components with the equipment shown in Fig. 10(b). There is no ambiguity in these results, and the equivalent rotation can easily be calculated from the phase data.

Because Polder's permeability matrix is made linear by approximations it is desirable to verify that the measurements made with circular polarization can be superposed to produce the same result as would be obtained using linear polarization.<sup>18</sup> For this purpose we choose a ferrite sample which exhibits a fairly complicated loss curve for both circular components. The absorption characteristics for both circular polariza-

<sup>18</sup> An experiment of this type was first performed by J. P. Schafer of Bell Telephone Laboratories in 1951 to establish the veracity of observed Faraday Rotations.

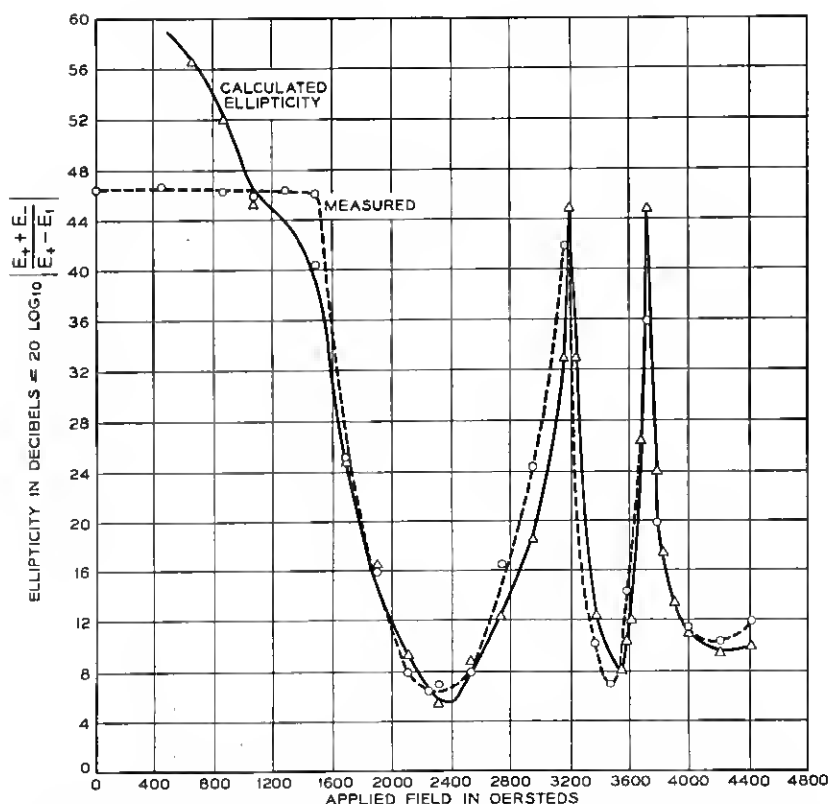


FIG. 12 — Ellipticity versus applied field as observed and as calculated from the data of Fig. 11.

tions are shown in Fig. 11. The resultant ellipticity in db is calculated from the relation

$$\Delta = 20 \log_{10} \left| \frac{E_+ + E_-}{E_+ - E_-} \right| \quad (18)$$

where  $E_{\pm}$  are the rms values of the circular components. This result is shown by the solid line in Fig. 12, and the ellipticity as measured by the set-up of Fig. 10(a) is shown by the dashed line. The agreement between these two curves is quite good if we consider that the accuracy of measuring each circular component is no better than  $\pm 0.05$  db so that in the regions where the two circular components are actually equal the calculated ellipticity can vary from infinity down to 52 db, while the signal to noise ratio of the equipment limits the measured ellipticity to 50 db.

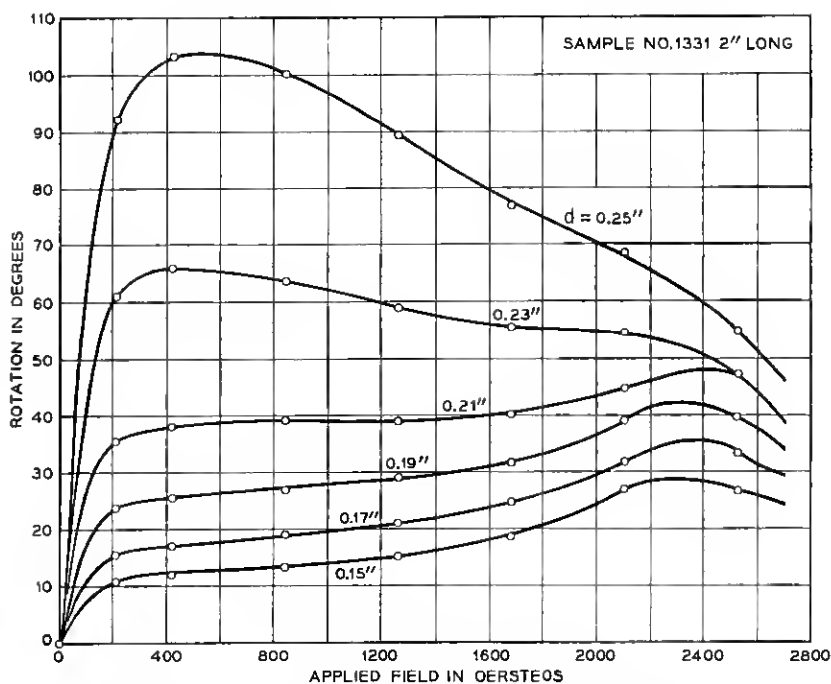


Fig. 13 — Dependence of rotation upon ferrite cylinder diameter.

To illustrate the effect of the diameter of the ferrite specimen upon the Faraday Rotation the following experiment was performed. A one-quarter inch cylinder suitably tapered on both ends was measured in the rotation measuring equipment and then ground down in steps of about 0.020" and remeasured after each cut. The particular ferrite chosen was one having very low dielectric loss and a dependence upon sample diameter which is typical of many ferrites. The data are shown in Fig. 13. At the largest diameter we observe a rotation which agrees with neither the plane wave theory nor the small perturbation approach. This behavior is evidently due to the presence of a higher order mode of propagation in the section of waveguide containing the ferrite cylinder.

Higher order mode effects are undesirable from the standpoint of bandwidth as the rotation produced thereby is much more frequency sensitive than that obtained from the  $TE_{11}$  mode. This is evident from consideration of the fact that the partially filled waveguide is much closer to the cutoff for the higher order modes than for the dominant mode, and we will show that frequency dependence is increased as waveguide approaches cut-off for a given mode.



## BANDWIDTH OF THE FARADAY ROTATION

Some consideration has been given to means of increasing the bandwidth over which the Faraday Rotation is relatively constant. Two possible ways of broadbanding the effect can be suggested on the basis of measurements and theory.

In the infinite medium-plane wave theory the Faraday Rotation is shown to be totally independent of frequency everywhere far above the ferromagnetic resonance frequency. One way of explaining this lack of dependence upon frequency is to observe that the rotation per unit wavelength *decreases* with increasing frequency while the number of wavelengths per unit length *increases* at the same rate so that the rotation per unit length remains constant. In a waveguide there are two effects which cause the rotation to be frequency dependent. In the first place the guide wavelength is not linearly related to frequency and secondly, where thin pencils of ferrite are used, the radial distribution of field varies with frequency in such a way as to add to the frequency dependence.

By surrounding the ferrite element with a material of the same dielectric constant as that of the ferrite the guide wavelength will be reduced to approximately one-fifth the cut-off wavelength and will be almost linearly related to the frequency in this region. Furthermore, the radial distribution will not change appreciably with frequency because, neglecting the difference in permeabilities, the waveguide is now filled with a uniform dielectric. On this same basis there should be no tendency for the structure to set up higher order modes, so that if care is taken in the design of the transitions from circular to rectangular waveguide, the higher order modes can be avoided.

While the above approach would give very good bandwidth a lack of very low-loss dielectrics having the proper values of dielectric constant limits its usefulness. One of the best dielectrics in this range is Micalex K, and it has a loss tangent of approximately 0.001 which would produce a total loss of several tenths of a db in a length of two or three inches.

The bandwidth can be increased in another way which requires no special dielectric materials. Suhl has shown that when the ferrite element is not perfectly matched to the waveguide the resulting multiple reflections can enhance or detract from the inherent rotation. When the element is an integral number of half wavelengths long the resulting rotation is maximum and when the element is an odd number of quarter-wavelengths long the rotation is minimum. Since the rotation of a perfectly matched cylinder increases with increasing frequency we must choose a length for the unmatched element such that it is an integral

number of half-wavelengths long at the low end of the frequency band in order to increase the rotation at that point. The ferrite chamber may be designed as a resonant cavity very tightly coupled so that over the band the input impedance satisfies the matching requirements even though there are multiple reflections inside the cavity.

Some ellipticity of the resultant wave will be introduced by this method as the wave will be linearly polarized only at those frequencies at which the ferrite element is an integral number of quarter-wavelengths long. However, if only a small correction is applied in this way the ellipticity will be tolerable.

The first and second broadbanding techniques may be combined in a manner which was discovered in the course of our measurements to determine bandwidth. Consider a circular waveguide such that at the low end of the band the wave is just slightly above cut-off. Then the wave impedance in the air-filled pipe will be high. If now the pencil of ferrite is supported in a polystyrene holder, the wave impedance in the ferrite region is much lower than that in the air filled region and multiple reflections will be set up. In order to reduce the magnitude of the reflections to achieve just the right degree of correction each end of the polystyrene can be tapered at the proper angle to give optimum correction. The polystyrene alone gives some improvement in bandwidth according to the arguments first submitted, and the internal reflections provide the rest of the compensation. The data shown in Fig. 14 show the variation of rotation with frequency obtained in this manner. The dotted extension of the low end of the curve indicates the expected rotation in the absence of the correction. By means of this compensation we are able to restrict the variation in rotation to  $\pm 4$  per cent over a band wider than 15 per cent. Over this band the ellipticity defined as the ratio in db of the maximum to the minimum field strength was over 50 db which corresponds to perfectly linear polarization within our ability to measure it. As a further advantage of this system the rotation obtained at all frequencies from the ferrite pencil is increased through the use of the polystyrene. This amplification of rotation follows directly from an extension of the plane theory.<sup>19</sup>

The rotation per unit length is given by the relation

$$\frac{\theta}{l} = \frac{\omega}{c} \sqrt{\epsilon} (\sqrt{\mu_-} - \sqrt{\mu_+}) \quad (19)$$

where  $\mu_{\pm}$  are the effective permeabilities seen by the circularly polarized

<sup>19</sup> Such an amplification of rotation was first observed and studied by J. P. Schafer of Bell Telephone Laboratories at Deal, N. J.

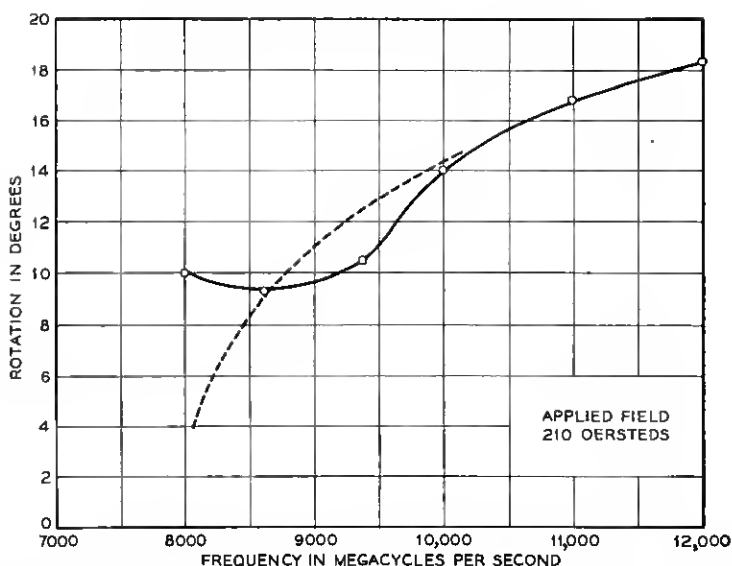


FIG. 14 — Variation of rotation with frequency showing broadbanding obtained through compensation technique.

components of the wave. The corresponding relation for the partially filled waveguide will be a transcendental expression involving  $\mu_{\pm}$  and  $\epsilon$  in a similar way so that arguments regarding the waveguide problem may be based upon equation 15 if we consider  $\mu_{\pm}$  and  $\epsilon$  to be effective values averaged over the guided mode. Thus by increasing the dielectric constant of the region surrounding the ferrite we increase the rotation by increasing the average value of  $\epsilon$ . Since we also change the radial distribution in such a way as to reduce the fraction of the power contained in the ferrite the amplification in rotation will be less than would be obtained if the waveguide diameter were reduced at the same time. In Fig. 16 we show the effect of increasing the dielectric constant of the region surrounding the ferrite and the effect of a subsequent reduction in guide size. Here again we see that if very high dielectric constants were available in low loss materials a really significant improvement in performance could be obtained. Nevertheless, even the effect of the polystyrene is useful and here we suffer a loss of less than 0.1 db at X-Band.

In the discussion of the loss mechanisms the second hump on absorption Curve F in Fig. 5 was described as a "cavity resonance". While the exact mode of resonance cannot be determined except from a complete solution of the partially filled waveguide problem, we are able to show that the subsidiary hump is strongly dependent upon the diameter of

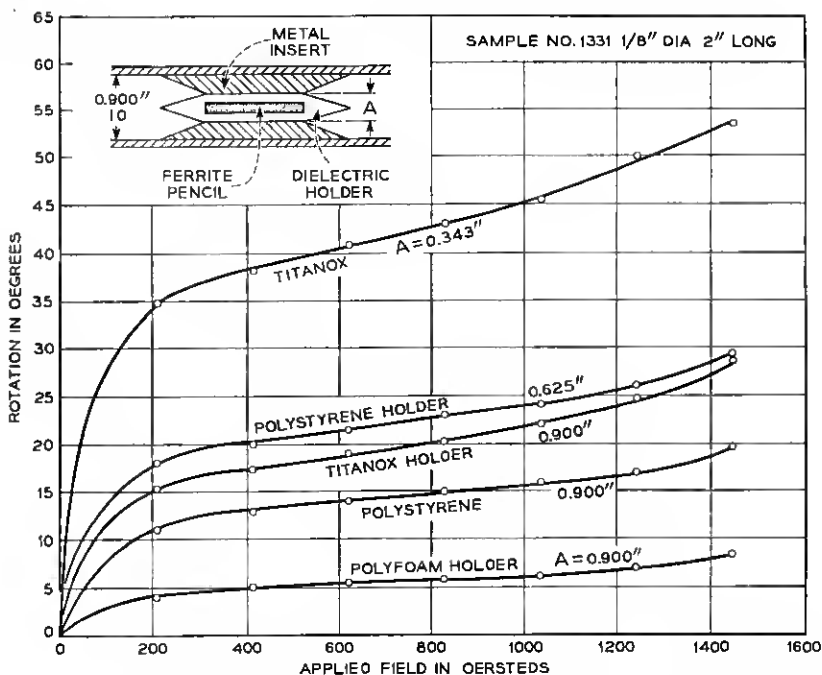


Fig. 15 — The dependence of the rotation characteristic upon the geometry and dielectric constant of the surrounding medium.

the ferrite element and upon the guide wavelength but that it is entirely independent of the total length of the element. In Fig. 16 we show the absorption of a positive circularly polarized wave as a function of applied field for several cylinder diameters. We see that the first hump remains relatively fixed in position while the second hump moves rapidly in the direction of higher fields and becomes larger in height as the diameter is increased. The first resonance is the ferromagnetic resonance, and its movement with increasing diameter is due simply to the fact that the demagnetizing factors of the cylinder are changed when the diameter is changed. The movement of the second hump can be explained if we assume that the resonance frequency of the shape resonance is given by an equation of the form

$$f = \frac{A}{d\sqrt{\mu'_+ \epsilon}} \quad (20)$$

This is an equation typical of those encountered in dimensional resonances. Here  $A$  is some constant associated with the geometry of the sample and  $d$  is the cylinder diameter. At a fixed frequency, we see that

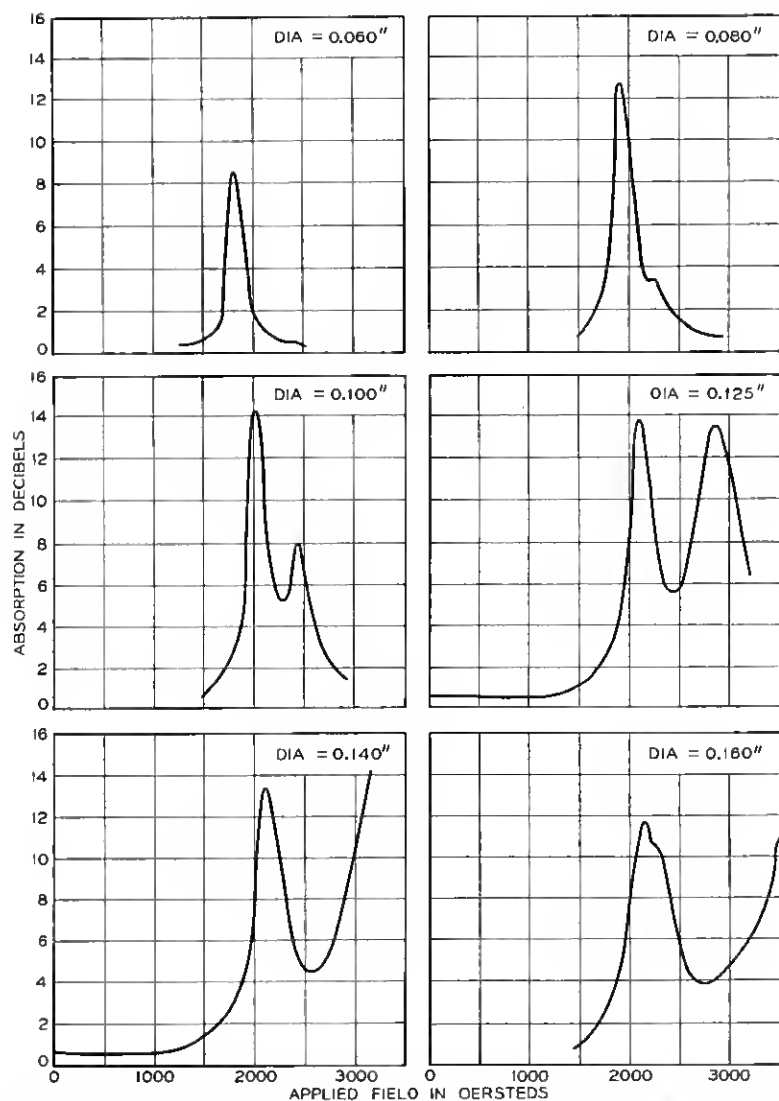


FIG. 16 — Absorption of a positive circularly polarized wave for several sample diameters showing "cavity resonances".

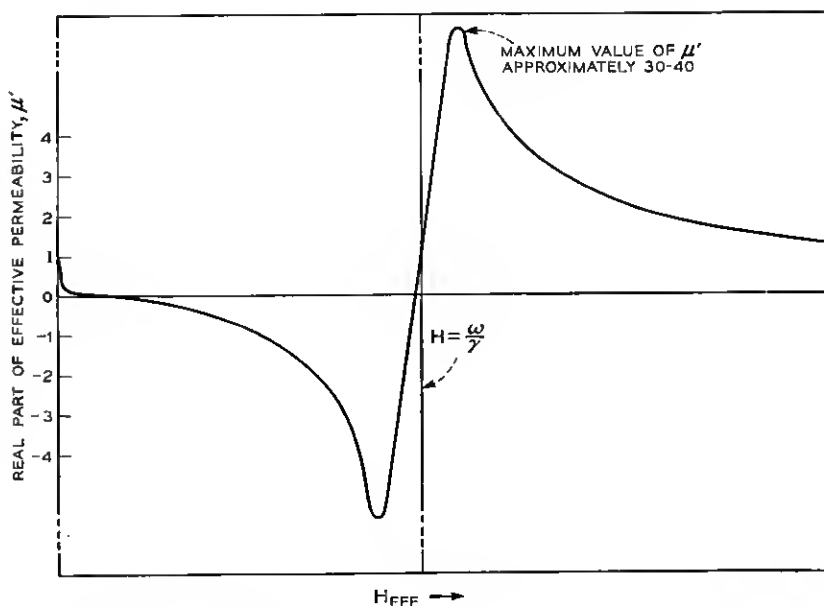


Fig. 17 — Exaggerated scale plot of the real part of the effective permeability versus effective dc magnetic field.

an increase in  $d$  must be accompanied by a decrease in  $\mu'_+$  to maintain resonance. For convenience we reproduce in Fig. 17 the behavior of the real part of the effective permeability as a function of field. We see that above resonance the permeability starts at a very high value and then decreases as we move toward higher fields. Thus as we increase the diameter,  $d$ , we would expect the "cavity" resonance to move to the right, and since in doing so we go further from the ferromagnetic resonance absorption we expect the height of the peak to increase due to the fact that the loss tangent of the permeability is smaller, and hence the effective  $Q$  of the "cavity resonance" is larger. Finally we observe that the broadening of the peak which occurs in the larger diameters is due simply to the fact that the slope of the  $\mu'_+$  curve decreases as we move to the right so that a larger change in dc field is required to take  $\mu'_+$  through a given variation.

A similar movement of the subsidiary hump can be effected through a change in the dielectric constant of the medium surrounding the ferrite element. However, altering the total length of the ferrite in steps of 0.030" until the total change was greater than 0.250" produced no noticeable effect upon either the relative heights or positions of the two peaks.

## FERRITE COMPOSITION AND IRON STOICHIOMETRY

There is a continuous program of measurement in which a large number of ferrites from various sources are tested at X-Band and other frequencies. In the early days of the program the most remarkable feature of the results was the tremendous variation in properties even among materials of supposedly the same composition. More recently some general trends have been observed, and it is now possible to correlate the microwave performance of certain types of ferrites with their chemical and structural nature. The nickel-zinc ferrites in particular have been studied extensively, and those having the approximate formula  $\text{Ni}_{0.3}\text{Zn}_{0.7}\text{Fe}_2\text{O}_4$  are quite good for microwave applications so long as they have slightly less iron than the above formula requires. A series of measurements illustrating the effect of slight variations in iron content yielded the data shown in Figs. 18 and 19. These ferrites were prepared by F. J. Schnettler of Bell Telephone Laboratories in such a way that their bulk properties correspond very closely to their crystallite properties. The correspondence between the bulk dc conductivity and the RF

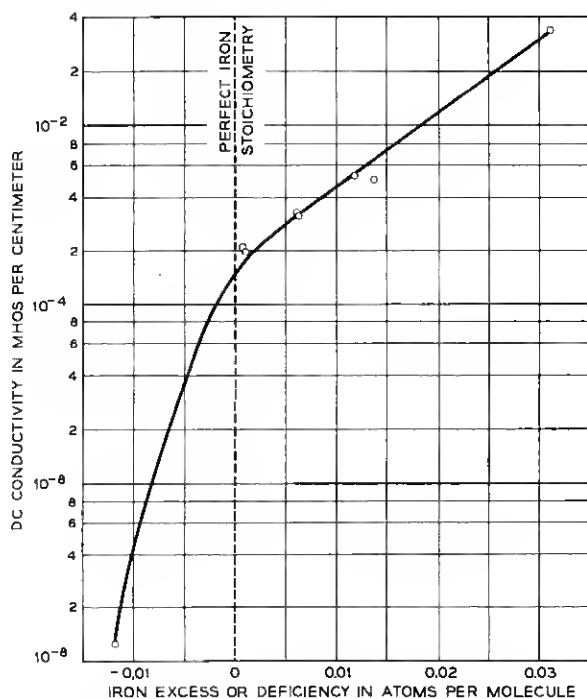


FIG. 18 — DC conductivity versus iron content of NiZn ferrite.

dielectric loss is particularly striking, indicating that the primary mechanism of loss in these ferrites is simply ohmic conductivity loss. This series of measurements clearly shows that the presence of even a very small amount of divalent iron is undesirable in ferrites to be used at microwave frequencies.

#### TRANSVERSE FIELD MEASUREMENTS

Two classes of measurements are commonly made in which a transverse dc magnetic field is applied parallel to the electric vector of the  $TE_{01}$  mode in rectangular waveguide. Those measurements involving a completely filled or symmetrically loaded waveguide yield primarily reciprocal effects which are useful in applications as previously mentioned. More useful, still, are the non-reciprocal effects observed when a piece of ferrite is placed asymmetrically in a waveguide and transversely mag-

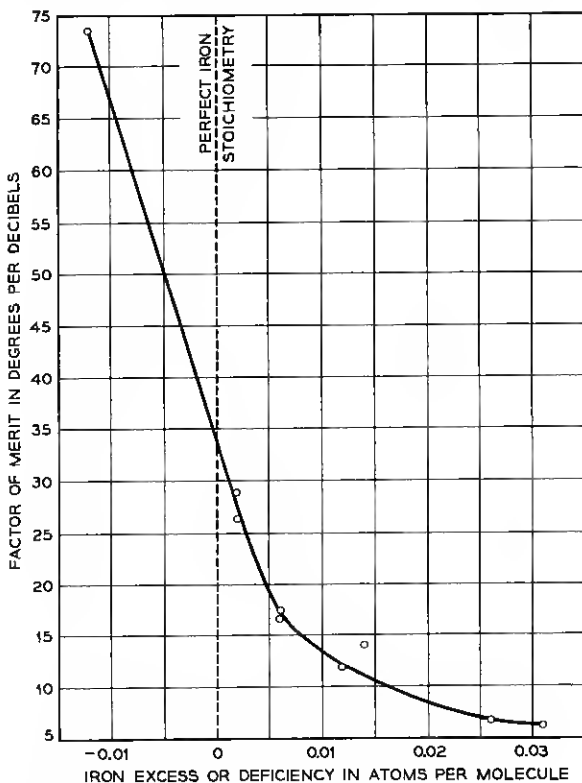


FIG. 19 — Factor of merit versus iron content of NiZn ferrite.



netized. Since these latter effects are quite new two measurements of this type are shown to illustrate the sort of performance obtainable. In the first experiment a slab of ferrite 20 mils thick was placed successively in several positions in a rectangular waveguide, and the phase shift as a function of field was measured for both directions of propagation. When the plate of ferrite is centrally located the phase shift is the same in both directions but when the ferrite is placed half way between the center and the edge of the guide a large difference in phase shift is observed. Finally when the slab of ferrite is located at the edge of the guide there is only a small difference between the positive and negative phase characteristics. This small difference will vanish as the thickness of the ferrite plate goes to zero. In Fig. 21 (a) we show the phase shift versus applied field for three positions of the ferrite slab and in Fig. 21 (b), the differential phase shift at a constant value of applied field is plotted against the position of the ferrite plate. The solid line curve taken at 9,500 mc indicates that maximum differential phase shift is obtained when the ferrite is located approximately 0.100" from the guide wall. Suhl's prediction is that the position at which maximum differential phase shift is observed should be independent of frequency. The dotted curve in Fig. 21 (b) taken at 8,200 mc verifies this part of the prediction in that the maximum again occurs where the ferrite plate is 0.100" from the guide wall even though the point at which  $h$  is circularly polarized has been shifted significantly by the change in frequency.

The second measurement was designed to measure the non-reciprocal absorption which is obtained when the strength of the dc magnetic field is adjusted so that the ferrite is at ferromagnetic resonance. In order to obtain the minimum forward loss and maximum reverse loss it is essential that the ferrite be located precisely at the point where the transverse and longitudinal components of the  $h$  field of the wave are equal, i.e., where the  $h$  field is circularly polarized in a plane perpendicular to the magnetic field. Since this condition exists at only one point in the half-waveguide the ferrite slab must be made very thin. Measurements were made in the 6000-7000 mc band in RG50 waveguide. A thin plate of "Ferramic G" was cut so as to extend from one broad wall to the other. Its length was approximately  $1\frac{1}{2}$ " and its thickness was originally 0.050" and was subsequently reduced to 0.025" and finally to 0.009". In the last case the ferrite was so fragile that it was necessary to support it by cementing it to a  $\frac{1}{8}$ -inch plate of polystyrene. For convenience the ferrite plate was fastened securely in place at a point calculated to be the point where the  $h$  vector is circularly polarized at the center of the band, and frequency was varied about this center frequency. At each

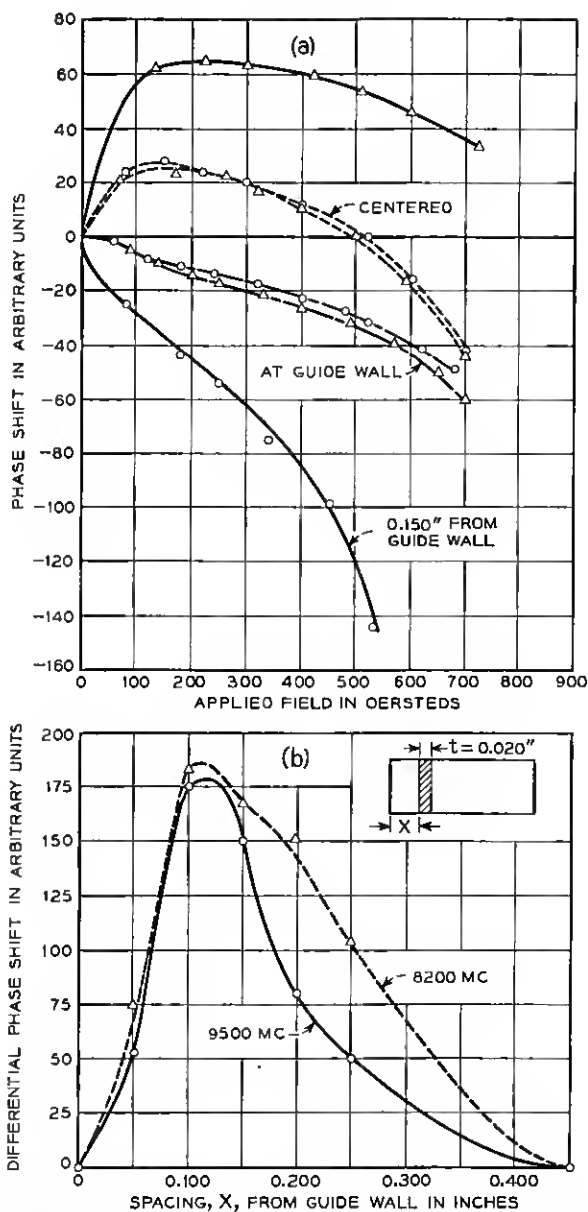


FIG. 20 — Phase shift versus applied field and differential phase shift versus position of the ferrite plate.

frequency the dc magnetic field was adjusted so that absorption was a maximum for the wave for which the absorption is the greater. The field was then reversed and the absorption was again measured. Since reversing the field is entirely equivalent to reversing the direction of propagation we obtain in this way the insertion loss for both directions of propagation. Data taken in this manner are shown in Fig. 22 along with a sketch of the geometry employed. The VSWR was measured in the case of the thinnest plate and was found to be less than 1.05 to 1 over the band. The variations in the dotted curve, however, are probably due to reflections from the ends of the sample.

One must bear in mind that as the frequency is changed, the field required for ferromagnetic resonance is changed so that these data do not give a true index of the bandwidth of the device. This must be measured at a constant value of field. However, if we had used a ferrite

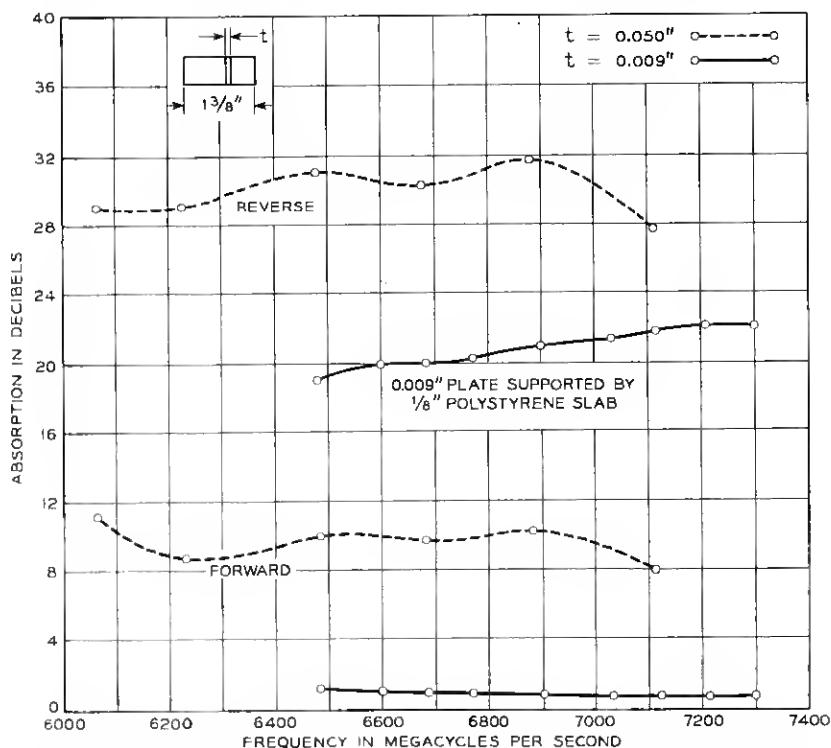


FIG. 21 — Absorption of forward and reverse waves at the optimum value of field as functions of frequency. Two sample thickness are shown to illustrate the effect of sample thickness.

having a loss characteristic such as is shown in Fig. 5, Curve C, in which the loss differential becomes great as soon as the material is saturated, then the bandwidth would be as shown in Fig. 21. Of all the materials which have been measured in the past two years only one sample has shown this effect. At the time such behavior was regarded as the antithesis of the ideal, and no further investigation was made. Now that there is a use for such a material, effort is being directed toward maximizing the effect. Since the effect is thought quite definitely to be due to a very high effective anisotropy field arising either from crystalline anisotropy or demagnetizing effects, the problem of creating the proper material should be quite straightforward.

At this point we should consider the relative merits of the transverse field non-reciprocal devices and those employing the Faraday rotation. We have seen that it is possible to construct a simple isolator using the non-reciprocal absorption in rectangular waveguide. Such an isolator has a minimum forward loss of more than 0.5 db for 20 db reverse loss where an isolator employing the Faraday Rotation can be made with less than 0.1 db forward loss for 30 db return loss. On the other hand the transverse field isolator is much simpler, more compact and easier to

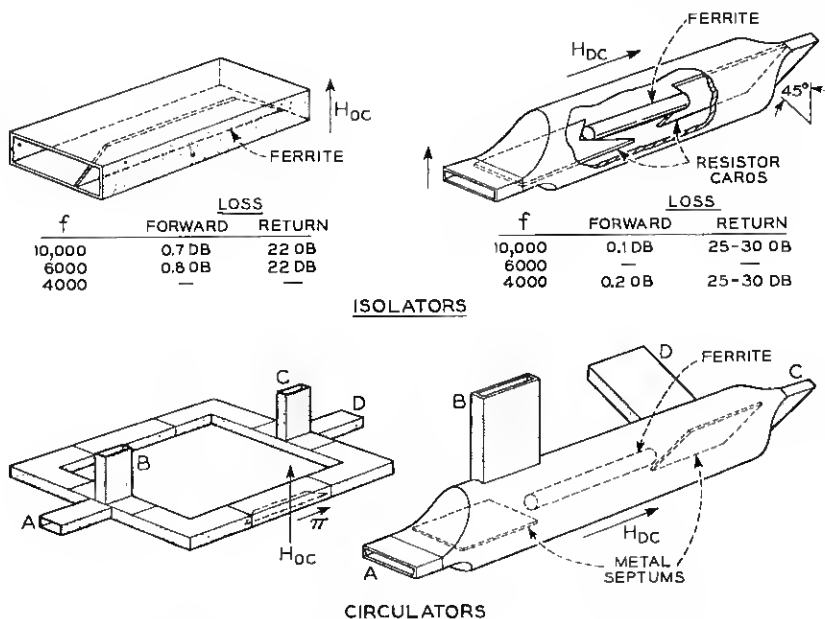


FIG. 22 — Comparison of two basic non-reciprocal elements of the Faraday effect and transverse field effect types.

match impedance-wise. To illustrate the comparison between comparable devices using each principle we have prepared a figure showing the relative structural complexity and performance standards of each. This comparison is shown in Fig. 22. In general the transverse field devices which depend on differential phase shift have about the same insertion loss as those employing the Faraday effect, while the transverse field devices which depend upon differential absorption are somewhat more lossy but simpler to construct than the corresponding Faraday Rotation devices.

#### SUMMARY

This paper has reviewed the plane wave theory, extended it to discuss waveguide effects, analyzed the various loss mechanisms present in ferrites at microwaves, and discussed numerous measurement techniques and results. It is known that there are original papers in preparation in Bell Telephone Laboratories, and possibly elsewhere, which make this review incomplete at the time of writing. However, it is hoped that the information summarized herein will be of assistance to those who are seeking orientation in this new and rapidly expanding field.

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